PFC/JA-92-4

Theory of Energetic Alpha particle Driven Alfven Wave Turbulence

Fong-Yan Gang

Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139
U. S. A.

This work was supported by the U.S. Department of Energy under contract No. DE-FG02-91ER-54109. Reproduction, translation, publication, use, and disposal, in whole or in part, by or for the U.S. Government is permited.

Submitted for publication in: Physics of Fluids B

Theory of Energetic Alpha Particle Driven Alfven Wave Turbulence

Fongyan Gang

Plasma Fusion Center

Massachusetts Institute of Technology

Cambridge, MA 02139

PACS Nos. 5235Ra,5235Mw,5230Dw,5225Fi

Abstract

A theory of energetic alpha particle driven Alfven wave turbulence is presented. Attention is focused on the nonlinear saturation of turbulence through bulk ion Compton scattering. It is found that the ion Compton scattering, while conserving total fluctuation energy, induces a nonlinear spectral shift away from the linear alpha particle growth rate towards higher poloidal wave number k_θ, thus reducing the power transfer between the energetic alpha particles and the Alfven wave turbulence. Saturation is obtained when this reduced alpha-Alfven wave power transfer is balanced by the linear electron Landau damping. The saturation level of the turbulence and the turbulent particle transport for both energetic alphas and bulk plasmas are calculated. The implications of these results on the alpha particle and plasma confinement are discussed.

I. Introduction

The alpha particles produced in a D-T fusion reaction carry a significant portion of total energy released. These energetic alpha particles can be confined by the strong magnetic field of a tokamak and transfer their energies to the bulk plasma through the classical collisional process. When the total fusion power released is large enough to offset the plasma heat loss, the plasma can burn on its own even without external heating (plasma ignition). This is the physics basis of thermal nuclear self-heating. However, the presence of energetic alpha particles can also introduce new types of plasma instabilities[1]. In particular, since the energetic alpha particles have a thermal speed v_α larger than the Alfven speed v_A: $v_\alpha > v_A$, shear Alfven wave instabilities can be excited via wave-particle resonance. The Alfven wave turbulence thus developed is expected to result in anomalous losses of the energetic alphas before they could be thermalized inside the plasma (loss of ignition capability). Furthermore, the Alfven wave turbulence may also enhance the bulk electron and ion transport, and lead to a degradation of bulk plasma confinement. Therefore, an understanding of the energetic alpha particle driven Alfven wave turbulence and associated plasma transport is very important to future burning plasma experiment.

The linear theory of alpha particles driven Alfven wave instabilities has been considered by many authors[2–5]. These analyses indicate that the shear Alfven waves can indeed be driven unstable by energetic alpha particles under certain conditions which are easily satisfied in future experimental devices. Previously, the nonlinear analyses of the problem were primarily in the area of particle loss through orbit overlap[4,6] and saturation of single mode via coherent wave trapping[7], which suffer either from non-self-consistency, or non-turbulent nature. More recently, the self-consistent turbulent treatments of the problem have been carried out[8,9] and saturation of turbulence due to quasi-linear alpha particle profile modification has been found in the case of no external particle source. However, in practical cases, due to the presence of external particle source (such as alpha particle production in fusion reaction), the alpha particle phase space gradients which are responsible for driving the Alfven wave instabilities may be maintained. In such case, the quasi-linear process may not be enough to saturate the Alfven wave turbulence and nonlinear effects of mode couplings have to be taken into account.

In this paper, we present a nonlinear turbulent theory of energetic alpha particle driven Alfven wave turbulence. We assume that the alpha particle phase space gradients are fixed by some external particle source, and the saturation of turbulence is through bulk ion Compton scattering (nonlinear interactions between Alfven wave turbulence and bulk ions), i.e. we assume that the nonlinear time scale is faster than the transport time scale. The reasons that we propose the ion Compton scattering as the dominant nonlinear saturation mechanism are based on the following considerations. Generally, there are two types of nonlinear mode couplings: kinetic type of nonlinear wave-particle interactions (Compton scattering), and fluid type of three wave resonant interactions. First of all, the wave-particle interactions are determined by their relative speeds. Since the electrons and energetic alpha particles move faster than the Alfven waves, i.e. $\omega = k_{\|} v_A \leq k_{\|} v_\alpha, k_{\|} v_e$, where v_e is the thermal speed of electrons, ω and $k_{\|}$ are the wave frequency and parallel wavenumber, they interact with the Alfven waves primarily through their linear wave-particle resonance. Their nonlinear wave-particle resonances (electron and alpha Compton scattering) represent higher order effects relative to their linear Landau resonances, and thus are negligible. For the ions, however, since they move much slower than the Alfven waves, their interaction with the Alfven waves is primarily through ion Compton scattering, i.e. $\omega = k_{\|} v_A > k_{\|} v_i$, but $\omega - \omega' \leq (k_{\|} - k'_{\|}) v_i$, where ($\omega$, $k_{\|}$) represents the test Alfven wave, while (ω', $k'_{\|}$) represents the turbulent bath. Second, for the Alfven wave instabilities driven by the alpha particles, the linear growth rate γ is much smaller than the wave frequency ω (weak turbulence regime). In this case, the fluid type three wave resonance interactions has been shown to be very weak[10] because the two fields $\tilde{\phi}$ (electrostatic potential) and $\tilde{\psi}$ (magnetic potential) are highly correlated due to the excitation of Alfven waves, i.e. $\tilde{\phi} = -v_A \tilde{\psi}$, which leads to cancellations of the fluid type nonlinear interactions. Therefore, we conclude that ion Compton scattering is the dominant nonlinear effect. Here, we like to point it out that ion Compton scattering of kinetic Alfven waves has been considered in the problem of nonlinear Alfven wave heating by Hasegawa and Chen [10]. The difference between their work and ours is that they consider nonlinear ion interaction with only *two* Alfven waves, while we consider nonlinear ion interaction with a spectrum of Alfven waves (turbulence). In their case, the ion Compton scattering due to parallel ion motion is dominant. However, in our case, this type ion Compton scattering is negligible because the interactions between ($\vec{k}$, ω) and ($\vec{k}'$, ω') for $\omega' > \omega$ and $\omega' < \omega$ cancel each other and lead to a negligible net effect. The dominant ion Compton scattering in our case comes from ion diamagnetic drift.

The remainder of this paper is organized as follows. In Sec.II, we derive a set of nonlinear equations that are suitable to describe the nonlinear evolution of Alfven wave turbulence. The essential kinetic effects such as linear electron and alpha particle Landau resonance, and ion Compton scattering, are included. In Sec.III, a nonlinear theory based on the ion Compton scattering is presented. A number of issues are discussed. They include the linear alpha growth rate with high k_θ cut off, the linear electron Landau damping rate, the nonlinear spectral transfer rate due to ion Compton scattering, the nonlinear steady state and saturation amplitudes, and anomalous particle transport for both the alpha particles and the bulk plasma. In Sec.IV, discussions and conclusions are presented.

II. Basic Model Equations

For a system of electrons, ions, and energetic alpha particles, we start with their corresponding nonlinear gyro-kinetic equations[11]:

$$-i(\omega - k_{\parallel}v_{\parallel} - \vec{k}_{\perp}\cdot\vec{v}_d^j)\tilde{g}^j_{\vec{k},\omega} = -i\frac{q_j}{T_j}f_0^j(\omega-\omega_j^*)\tilde{\varphi}_{\omega,\vec{k}}J_0(\frac{k_{\perp}v_{\perp}}{\Omega_j}) + N^j_{\vec{k},\omega} \qquad (1)$$

where the nonlinearity $N^j_{\vec{k},\omega}$ is given by:

$$N^j_{\vec{k},\omega} = \frac{c}{B}\sum_{\substack{\vec{k}'\\ \omega'}}(\vec{e}_{\parallel}\times\vec{k}'\cdot\vec{k})\tilde{\varphi}_{\vec{k}',\omega'}\tilde{g}^j_{\vec{k}-\vec{k}',\omega-\omega'}J_0(\frac{k'_{\perp}v_{\perp}}{\omega_j}) \qquad (2)$$

In the above equations, j=e, i, α denotes species, q_j is the particle charge, T_j is the particle temperature, c is the speed of light, B is the magnetic field strength, $\tilde{\varphi} = \tilde{\phi} - (v_{\parallel}/c)\tilde{\psi}$, $\tilde{\phi}$ is the electrostatic potential, $\tilde{\psi}$ is the magnetic potential, J_0 is Bessel function of zeroth order, $\vec{e}_{\parallel}$ is the unit vector in the direction of equilibrium magnetic field, $\tilde{g}^j_{\vec{k},\omega}$ is the nonadiabatic part of the perturbed particle distribution function $\tilde{f}^j_{\vec{k},\omega}$:

$$\tilde{f}^j_{\vec{k},\omega} = -\frac{q_j}{T_j}\tilde{\phi}_{\vec{k},\omega}f_0^j + \tilde{g}^j_{\vec{k},\omega}e^{i\frac{\vec{e}_{\parallel}\cdot\vec{k}_{\perp}\times\vec{v}_{\perp}}{\Omega_j}} \qquad (3)$$

f_0^j is the equilibrium particle distribution, and is assumed to be a Maxwelliam for simplicity:

$$f_0^j = N_j(\frac{m_j}{2\pi T_j})^{\frac{3}{2}}e^{-\frac{m_j v^2}{2T_j}}$$

m_j is the particle mass, N_j is the particle density, $\Omega_j = (q_jB/m_jc)$ is the particle gyro-frequency, $\omega_j^* = (cT_j/Bq_j)\vec{k}\cdot\vec{e}_{\parallel}\times\vec{\kappa}_n$ is the particle diamagnetic drift frequency,

$\vec{\kappa}_n^j = \vec{\nabla} \ln f_0^j$ is the particle density gradient vector, and $\vec{v}_d^j = ([v_\parallel^2 + (v_\perp^2/2)]/\Omega_j)\vec{e}_\parallel \times \vec{\kappa}_c$ is the particle curvature drift velocity, and $\vec{\kappa}_c = \vec{\nabla} \ln B$ is the magnetic field gradient vector.

Since our objective is to gain physical insight, the calculations are carried out in a simple shearless slab geometry. For each species, the particle density, parallel current, and pressure perturbation which correspond to the zeroth, first, and second moment of the distribution function $\tilde{f}^j_{\vec{k},\omega}$ can be obtained:

$$\tilde{n}^j_{\vec{k},\omega} = -\frac{q_j}{T_j} N_j \tilde{\phi}_{\vec{k},\omega} + \int d^3\vec{v}\tilde{g}^j_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_j}) \tag{4}$$

$$\tilde{J}^j_{\parallel\vec{k},\omega} = q_j \int d^3\vec{v} v_\parallel \tilde{g}^j_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_j}) \tag{5}$$

$$\tilde{p}^j_{\vec{k},\omega} = \int d^3\vec{v} m_j (v_\parallel^2 + \frac{v_\perp^2}{2}) \tilde{g}^j_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_j}) \tag{6}$$

From the quasi-neutrality condition: $\sum_j q_j \tilde{n}^j_{\vec{k},\omega} = 0$, we have:

$$\sum_j q_j \int d^3\vec{v}\tilde{g}^j_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_j}) = \sum_j N_j \frac{q_j^2}{T_j} \tilde{\phi}_{\vec{k},\omega} \tag{7}$$

Multiplying Eq.(1) on both sides with $q_j J_0(k_\perp v_\perp/\Omega_j)$, integrating over velocity space $\int d^3\vec{v}$, and summing over the species j, we have:

$$\begin{aligned} &-i\omega\tilde{\phi}_{\vec{k},\omega} \sum_j N_j \frac{q_j^2}{T_j} + ik_\parallel \tilde{J}_{\parallel\vec{k},\omega} + i\frac{c}{B}(\vec{e}_\parallel \times \vec{\kappa}_c \cdot \vec{k})\tilde{p}_{\vec{k},\omega} \\ =& -i\omega\tilde{\phi}_{\vec{k},\omega} \sum_j N_j \frac{q_j^2}{T_j}(1 - \frac{\omega_j^*}{\omega})\Gamma_0(b_j) + \sum_j q_j \int d^3\vec{v} N^j_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_j}) \end{aligned} \tag{8}$$

where Eqs.(5)-(7) have been used. In the above equation, $\tilde{J}_{\parallel\vec{k},\omega} = \sum_j \tilde{J}^j_{\parallel\vec{k},\omega}$ is the total parallel current, $\tilde{p}_{\vec{k},\omega} = \sum_j \tilde{p}^j_{\vec{k},\omega}$ is the total plasma pressure, $\Gamma_0(b_j) =$

$I_0(b_j)e^{-b_j}$, $I_0(b_j)$ is the modified Bessel function of zeroth order, $b_j = (1/2)k_\perp^2\rho_j^2$, $\rho_j = v_j/\Omega_j$ is the particle gyroradius, and v_j is the particle thermal speed.

Our basic model equations are derived from Eqs.(7) and (8). A number of approximations can be made. First, since the alpha particle is a minority species but with a higher temperature, i.e. $N_i \sim N_e \gg N_\alpha$, but $T_i \sim T_e \ll T_\alpha$, we have $N_i/T_i \sim N_e/T_e \gg N_\alpha/T_\alpha$, but $N_iT_i \sim N_eT_e \sim N_\alpha T_\alpha$. Thus, we need only to keep the alpha particle contribution to the third term of Eq.(8) which is proportional to the alpha particle pressure. Second, for the electrons and ions, we have $b_e \ll b_i \ll 1$, and ω_i^*, $\omega_e^* \ll \omega$. Using $\Gamma_0(b_i) \simeq 1 - b_i$, and $\Gamma_0(b_e) \simeq 1$, Eq.(8) reduces to:

$$i\omega\tilde{\phi}_{\vec{k},\omega}N_i\frac{q_i^2}{T_e}(-k_\perp^2\rho_s^2) + ik_\parallel\tilde{J}_{\parallel\vec{k},\omega} + i\frac{c}{B}(\vec{e}_\parallel \times \vec{\kappa}_c \cdot \vec{k})\tilde{p}_{\vec{k},\omega}$$
$$= \sum_{j=i,e} q_j \int d^3\vec{v}N^j_{\vec{k},\omega}J_0(\frac{k_\perp v_\perp}{\Omega_j}) \qquad (9)$$

In the above equation, the nonlinear terms represent mode couplings and generally can be divided into two parts: fluid part and kinetic part. The fluid part comes from three wave resonant interactions, and is modeled by the nonlinearities in MHD (magnetohydrodynamics) turbulence. The kinetic part comes from nonlinear wave-particle interactions (Compton scattering). Since electron moves much faster than the Alfven waves, its Compton scattering only constitutes a higher order correction to its linear Landau damping, and thus is negligible. However, since ion moves much slower than the Alfven waves, the linear ion Landau damping is negligible, and the ion Compton scattering which represents a higher order effect has to be kept. Based on these considerations, the nonlinear terms in Eq.(9) can be symbolically written as:

$$\sum_j q_j \int d^3\vec{v} N^j_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_j}) = (\mathrm{MHD})_{\mathrm{nl}} + \mathrm{N_i}\frac{\mathrm{q_i^2}}{\mathrm{T_e}}\Pi^{\mathrm{i}}_{\vec{\mathrm{k}},\omega} \tag{10}$$

where $(\mathrm{MHD})_{\mathrm{nl}}$ represents the MHD nonlinearities which will be given later, $\Pi^i_{\vec{k},\omega}$ is a fluid representation of the ion Compton scattering. Since the growth rate of the instability γ is much smaller than the mode frequency ω, $\Pi^i_{\vec{k},\omega}$ can be obtained following the standard procedure of weak turbulence theory[12]:

$$N_i \frac{q_i^2}{T_e}\Pi^i_{\vec{k},\omega} = q_i \int d^3\vec{v} N^{i(3)}_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_i}) \tag{11}$$

where

$$N^{i(3)}_{\vec{k},\omega} = \frac{c}{B}\sum_{\substack{\vec{k}' \\ \omega'}} (\vec{e}_\parallel \cdot \vec{k}' \times \vec{k})\tilde{\varphi}_{\vec{k}',\omega'}\tilde{g}^{i(2)}_{\vec{k}-\vec{k}',\omega-\omega'} J_0(\frac{k'_\perp v_\perp}{\Omega_i}) \tag{12}$$

and $\tilde{g}^{i(2)}_{\vec{k}-\vec{k}',\omega-\omega'}$ is determined by the direct beating between the test mode $(\vec{k},\,\omega)$ and turbulent bath $(\vec{k}',\,\omega')$. From Eq.(1), we have:

$$\begin{aligned}\tilde{g}^{i(2)}_{\vec{k}-\vec{k}',\omega-\omega'} =& \frac{c}{B} R^i_{\vec{k}-\vec{k}',\omega-\omega'}(\vec{e}_\parallel \cdot \vec{k}' \times \vec{k})[\tilde{\varphi}_{\vec{k},\omega}\tilde{g}^{i(1)}_{-\vec{k}',-\omega'} J_0(\frac{k_\perp v_\perp}{\Omega_i}) \\ & -\tilde{\varphi}_{-\vec{k}',-\omega'}\tilde{g}^{i(1)}_{\vec{k},\omega} J_0(\frac{k'_\perp v_\perp}{\Omega_i})]\end{aligned} \tag{13}$$

where $R^i_{\vec{k},\omega} = i/(\omega - v_\parallel k_\parallel - \vec{v}^i_d \cdot \vec{k})$ is the unperturbed particle propagator, and $\tilde{g}^{i(1)}_{\vec{k},\omega}$ is the linear ion response:

$$\tilde{g}^{i(1)}_{\vec{k},\omega} = -i\frac{q_i}{T_i} f^i_0 R^i_{\vec{k},\omega}(\omega - \omega^*_i)\tilde{\varphi}_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_i}) \tag{14}$$

Substituting Eqs.(13) and (14) into Eq.(12), we have:

$$N_i \Pi^i_{\vec{k},\omega} = i(\frac{c}{B})^2 \frac{T_e}{T_i} \int d^3\vec{v} f_0^i \sum_{\substack{\vec{k}' \\ \omega'}} (\vec{e}_\| \cdot \vec{k}' \times \vec{k})^2 R^i_{\vec{k}-\vec{k}',\omega-\omega'} \tilde{\varphi}_{\vec{k},\omega} |\tilde{\varphi}_{\vec{k}',\omega'}|^2$$

$$\times J_0^2(\frac{k_\perp v_\perp}{\Omega_i}) J_0^2(\frac{k'_\perp v_\perp}{\Omega_i}) [R^i_{\vec{k},\omega}(\omega - \omega_i^*) - R^i_{\vec{k}',\omega'}(\omega' - \omega_i^{*\prime})] \tag{15}$$

Now, replacing $k_\perp^2$ by $-\nabla_\perp^2$, $-i\omega$ by $\partial/\partial t$, Eq.(9) then becomes:

$$N_i \frac{q_i^2}{T_e} \{ \frac{\partial}{\partial t}(\rho_s^2 \nabla_\perp^2 \tilde{\phi}) + \frac{c}{B} \vec{e}_\| \times \vec{\nabla}\tilde{\phi} \cdot \vec{\nabla}(\rho_s^2 \nabla_\perp^2 \tilde{\phi}) + \tilde{\Pi}^i \}$$

$$= (\vec{e}_\| + \frac{1}{B} \vec{\nabla}\tilde{\psi} \times \vec{e}_\|) \cdot \vec{\nabla} \tilde{J}_\| + \frac{c}{B} \vec{e}_\| \times \vec{\kappa}_c \cdot \vec{\nabla}\tilde{p} \tag{16}$$

where the second and fifth term are the MHD nonlinearities which are denoted by $(\mathrm{MHD})_{\mathrm{nl}}$ in Eq.(10).

The second equation that relates $\tilde{\phi}$ and $\tilde{\psi}$ can be obtained from the quasi-neutrality condition Eq.(7). In deriving such an equation, we will negelect all the kinetic effects for purpose of easy manipulations. The result is equivalent to the ideal MHD condition $E_\| = 0$, i.e.

$$\frac{\partial}{\partial t}\psi + \frac{c}{B} \vec{e}_\| \times \vec{\nabla}\tilde{\phi} \cdot \vec{\nabla}\psi = -c \vec{e}_\| \cdot \vec{\nabla}\tilde{\phi} \tag{17}$$

This approximation is justified for the following two reasons: (1) Including the kinetic effects of wave-particle resonance in Eq.(17) does not produce any new and significant physics. Specifically, the effects of wave-particle resonance if included are smaller than those in Eq.(16) by a small factor $k_\perp^2 \ll 1$. (2) The parallel electric field $E_{\parallel} = -\vec{e}_{\parallel} \cdot \vec{\nabla}\nabla_{\perp}^2 \phi$ induced by ion FLR effect is small for $k_{\perp}^2 \ll 1$. The energy exchange between wave and particles is primarily determined by particle curvature drift $\vec{v}_d^j$ and the perpendicular electric field $\vec{E}_{\perp}$.

Eqs.(16) and (17) constitute our basic model equations. To facilitate the analysis, we normalize the spatial and temporal variables $\vec{x}$, and t to units of ρ_s, and Ω_i^{-1} respectively, velocity variable to c_s, the electric potential $\tilde{\phi}$ to T_e/e, the magnetic potential $\tilde{\psi}$ to $\rho_s B$, and the particle pressure to $N_e T_e$. The normalized model equations then take the form:

$$\frac{\partial}{\partial t}\tilde{U} + \vec{e}_{\parallel} \times \vec{\nabla}\tilde{\phi} \cdot \vec{\nabla}\tilde{U} + \tilde{\Pi}^i = v_A^2 (\vec{e}_{\parallel} + \vec{\nabla}\tilde{\psi} \times \vec{e}_{\parallel}) \cdot \vec{\nabla}\tilde{J}_{\parallel} + \vec{e}_{\parallel} \times \vec{\kappa}_c \cdot \vec{\nabla}\tilde{p} \tag{18}$$

$$\frac{\partial}{\partial t}\psi + \vec{e}_{\parallel} \times \vec{\nabla}\tilde{\phi} \cdot \vec{\nabla}\tilde{\psi} = -\vec{e}_{\parallel} \cdot \vec{\nabla}\tilde{\phi} \tag{19}$$

where $\tilde{U}$ is the fluid vorticity: $\tilde{U} = \nabla_{\perp}^2 \tilde{\phi}$, and the parallel current $\tilde{J}_{\parallel} = -\nabla_{\perp}^2 \tilde{\psi}$ is obtained from the parallel Faraday's Law. The above equations describe the nonlinear evolutions of shear Alfven waves and contain all the essential kinetic effects, namely, the electron and alpha particle Landau resonance ($\tilde{p}$), and the ion Compton scattering ($\tilde{\Pi}^i$). The shear Alfven waves can be resonantly destabilized and develop into Alfven wave turbulence. The nonlinear evolutions of the Alfven wave turbulence in the presence of bulk ion Compton scattering will be discussed next.

III. Nonlinear Theory

The model equations (18) and (19) have two type nonlinearities: fluid nonlinearities represented by terms $(\vec{e}_{\parallel} \times \vec{\nabla}\tilde{\phi} \cdot \vec{\nabla}\tilde{U} - v_A^2 \vec{\nabla}\tilde{\psi} \times \vec{e}_{\parallel} \cdot \vec{\nabla}\tilde{J}_{\parallel})$ in Eq.(18), term $(\vec{e}_{\parallel} \times \vec{\nabla}\tilde{\phi} \cdot \vec{\nabla}\tilde{\psi})$ in Eq.(19), and kinetic nonlinearity represented by $\tilde{\Pi}^i$ in Eq.(18). For alpha particle driven Alfven wave instabilities, the growth rate γ is much smaller than the mode frequency ω (weak turbulence regime), the effect of fluid nonlinearities has been shown to be very small[8]. Qualitatively, this can be understood in the following way. In the weak turbulence regime, the two fields $\tilde{\phi}$ and $\tilde{\psi}$ are highly correlated due to the excitation of Alfven waves. From Eq.(19), we have $\tilde{\psi} \simeq (k_{\parallel}/\omega)\tilde{\phi}$. For the unstable kinetic Alfven wave: $\omega = -v_A k_{\parallel}$, the relation between $\tilde{\phi}$ and $\tilde{\psi}$ then becomes: $v_A\tilde{\psi} = -\tilde{\phi}$. If we substitute this relation into Eqs.(18) and (19), we find that the fluid nonlinearities cancel out with each other completely or up to an order of $k_{\perp}^2$ if ion finite Larmor radius (FLR) effect is taken into account. As a result, the fluid nonlinear interactions (MHD turbulence) are smaller than the ion Compton scattering at least by an order of $k_{\perp}^2$. For alpha particle driven Alfven waves the typical wavenumber $k_{\perp}$ is approximately given by $1/\rho_{\alpha}$ in the slab geometry (even smaller in a torus), thus $k_{\perp}^2 \simeq (\rho_s/\rho_{\alpha})^2 \ll 1$. Therefore, for the problem concerned, ion Compton scattering is the dominant nonlinear process.

Now, we neglect the fluid nonlinearities in both Eqs.(18) and (19). Fourier transforming to $(\vec{k},\ \omega)$ space, Eqs.(18) and (19) can be combined into a single nonlinear eigenmode equation:

$$\{\omega^2 - k_\parallel^2 v_A^2 - i\frac{\omega}{k_\perp^2}\delta^{nl}_{\vec{k},\omega} - \frac{\omega}{k_\perp^2}(\vec{e}_\parallel \times \vec{\kappa}_c \cdot \vec{k}_\perp)\delta^{l}_{\vec{k},\omega}\}\tilde{\phi}_{\vec{k},\omega} = 0 \tag{20}$$

where $\delta^{nl}_{\vec{k},\omega}$ and $\delta^{l}_{\vec{k},\omega}$ are the nonlinear ion response and linear plasma pressure response respectively :

$$\tilde{\Pi}^i_{\vec{k},\omega} = \delta^{nl}_{\vec{k},\omega}\tilde{\phi}_{\vec{k},\omega} \tag{21}$$

$$\tilde{p}_{\vec{k},\omega} = \delta^{l}_{\vec{k},\omega}\tilde{\phi}_{\vec{k},\omega} \tag{22}$$

which will be calculated from the kinetic theory.

Eq.(20) can be solved perturbatively. To the lowest order, we have the linear shear Alfven wave: $\tilde{\phi}_{\vec{k},\omega} = \tilde{\phi}_{\vec{k}}\delta(\omega - \omega_{\vec{k}})$, with the mode frequency $\omega_{\vec{k}}$ and linear growth rate $\gamma^l_{\vec{k}}$ given by:

$$\omega_{\vec{k}} = \pm v_A k_\parallel \tag{23}$$

$$\gamma^l_{\vec{k}} = \frac{\vec{e}_\parallel \times \vec{\kappa}_c \cdot \vec{k}_\perp}{2k_\perp^2} Im\delta^l_{\vec{k}} \tag{24}$$

To the next order, multiplying Eq.(20) by $\tilde{\phi}^*_{\vec{k},\omega}$, taking the imaginary part, and replacing γ by $(1/2)(\partial/\partial t)$, we obtain a wave-kinetic equation that describe the nonlinear evolution of Alfven wave spectrum in the presence of ion Compton scattering:

$$\frac{\partial}{\partial t}|\tilde{\phi}|^2_{\vec{k}} = 2(\gamma^l_{\vec{k}} + \gamma^{nl}_{\vec{k}})|\tilde{\phi}|^2_{\vec{k}} \tag{25}$$

where γ_i^{nl} is the nonlinear transfer rate due to ion Compton scattering:

$$\gamma_{\vec{k}}^{nl} = \frac{1}{2k_\perp^2} Re\delta_{\vec{k}}^{nl} \tag{26}$$

Eq.(25) is equivalent to an energy conservation Law. If we define total fluid energy as:

$$\begin{aligned} E^f =& \frac{1}{2}\int d^3\vec{x}[(\vec{\nabla}_\perp\tilde{\phi})^2 + v_A^2(\vec{\nabla}_\perp\tilde{\psi})^2] \\ =& \sum_{\vec{k}} k_\perp^2 |\tilde{\phi}|_{\vec{k}}^2 \end{aligned} \tag{27}$$

which is the sum of fluid convection energy and magnetic fluctuation energy. The time evolution of E^f is given by:

$$\frac{\partial}{\partial t}E^f = \sum_{\vec{k}} 2(\gamma^l + \gamma_i^{nl})k_\perp^2|\tilde{\phi}|_{\vec{k}}^2 \tag{28}$$

i.e. the nonlinear evolution of Alfven wave turbulence is determined by the α particle growth and bulk plasma dissipation. At steady state (or saturation), we have $\partial E^f/\partial t = 0$, Eq.(26) then indicates that the saturated state can be obtained either by reducing the linear growth rate γ^l (quasi-linear profile modification) or by reducing the overlap between the linear growth rate and fluctuation spectrum in wavenumber space (nonlinear saturation). Physically, a nonlinear spectral shift away from the linear growth rate reduces the power transfer between alpha particles and the Alfven wave turbulence. In the following, we assume that the α particle phase space gradients are fixed by some sort of external particle source and consider the saturation of plasma turbulence through ion Compton scattering. We proceed to calculate the linear growth rate $\gamma_{\underline{k}}^{l}$ and ion Compton scattering rate $\gamma_{\underline{k}}^{nl}$.

a. Linear Growth Rate

The linear stability analyses of shear Alfven waves in the presence of energetic Alpha particles have been studied by many authors[2-4]. However, most of the studies limit themselves to the case of low n or $k_\theta \rho_\alpha < 1$, where n is the toroidal mode number. In this regime, it is found that the linear growth rate increases linearly with k_θ. More recently, the stability of high n TAE mode has been considered by Berk et al[13]. They found that the linear growth rate or wave-particle power transfer is significantly reduced for large $k_\theta \rho_\alpha$ because the radial mode width becomes much smaller than the particle orbit width in such regime.

Since the linear growth rate, especially its dependence on k_θ, is very important for our nonlinear analysis, we will reproduce the derivation of the linear growth rate here in a slab geometry. In particular, we will show that the decay of the linear growth rate with increasing $k_\theta \rho_\alpha$ for $k_\theta \rho_\alpha > 1$ is due to the fact that the perpendicular curvature drift resonance rather than the parallel transit resonance becomes dominant. We will calculate the alpha particle growth rate first, the electron Landau damping rate can be calculated in the same way.

From Eq.(1), we can obtain the linear perturbed α particle distribution function

$\tilde{g}^{\alpha}_{\vec{k},\omega}$. In normalized units, we have:

$$\tilde{g}^{\alpha}_{\vec{k},\omega} = -i\frac{q_\alpha}{T_\alpha} f_0^\alpha R^{\alpha}_{\vec{k},\omega}(\omega - \omega^*_\alpha)\tilde{\varphi}_{\vec{k},\omega} J_0(\frac{k_\perp v_\perp}{\Omega_\alpha}) \tag{27}$$

where $\tilde{\varphi}_{\vec{k},\omega} = [1 - (k_\| v_\| /\omega)]\tilde{\phi}_{\vec{k},\omega}$, q_α is normalized to e, m_α is normalized to m_i, and T_α is normalized to T_e. The α particle pressure response $\tilde{p}^{\alpha}_{\vec{k},\omega}$ can be calculated from Eq.(6). Combining Eqs.(22) and (24), the linear α particle growth rate is:

$$\gamma^l_{\alpha\vec{k}} = -\frac{\pi}{2k_\perp^2}\frac{q_\alpha^2}{T_\alpha}(1 - \frac{\omega^*_\alpha}{\omega})\int d^3\vec{v}(\vec{v}^\alpha_d \cdot \vec{k})^2 f_0^\alpha \delta(\omega - k_\| v_\| - \vec{v}^\alpha_d \cdot \vec{k}_\perp) J_0^2(\frac{k_\perp v_\perp}{\Omega_\alpha}) \tag{30}$$

From the above equation, we note that: (1) alpha growth rate is positive for $\omega^*_\alpha > \omega$, i.e. only one branch of the shear Alfven wave $\omega_{\vec{k}} = -v_A k_\|$ is unstable. (2) the wave particle resonance is described by: $\omega - k_\| v_\| - \vec{k}_\perp \cdot \vec{v}^\alpha_d = 0$. Since the ratio $\vec{v}^\alpha_d \cdot \vec{k}_\perp / v_\| k_\| \sim v_\alpha \rho_\alpha k_\perp \kappa_c / v_\alpha k_\| \sim k_\perp \rho_\alpha$, the wave-particle interaction is dominated by particle parallel transit resonance: $\omega - k_\| v_\| \simeq 0$ for $k_\perp \rho_\alpha < 1$, and by particle perpendicular curvature drift resonance: $\omega - \vec{v}^\alpha_d \cdot \vec{k}_\perp \simeq 0$ for $k_\perp \rho_\alpha > 1$. Let's look at these two cases separately.

In the case of $k_\perp \rho_\alpha < 1$, the Bessel function in Eq.(30) can be approximated by 1. Carrying out the velocity integration, we get

$$\frac{\gamma^l_{\alpha\vec{k}}}{\omega} = -\frac{\sqrt{\pi}}{2k_\perp^2} N_\alpha \frac{q_\alpha^2}{T_\alpha}(1 - \frac{\omega^*_\alpha}{\omega})(\frac{\omega^\alpha_d}{\omega})^2 F(\xi_\alpha) \tag{31}$$

where $F(\xi_\alpha) = \xi_\alpha(\xi_\alpha^4 + \xi_\alpha^2 + 1/2)\exp\{-\xi_\alpha^2\}$ with $\xi_\alpha = \omega/|k_\| v_\alpha|$ is the fraction of energetic *alpha* particles that are in resonance with the Alfven wave, $\omega^\alpha_d =$

$(v_\alpha^2/\Omega_\alpha)\vec{e}_\parallel \times \vec{\kappa}_c \cdot \vec{k}$ is the curvature drift frequency for thermal alphas. Eq.(31) indicates that the alpha particle growth rate increases linearly with poloidal wave number $k_\theta \rho_\alpha$ for $k_\perp \rho_\alpha < 1$.

In the case of $k_\perp \rho_\alpha > 1$, the calculation is somewhat complicated. In order to carry out the velocity integration, we make the following transformation: $y_\perp = v_\perp / v_\alpha$, and $\epsilon = (v_\parallel^2 + v_\perp^2/2)/v_\alpha^2$. Carrying out the ϵ integration by using the δ function in Eq.(30), we obtain:

$$\frac{\gamma_\alpha^l}{\omega} = -\frac{\sqrt{\pi}}{k_\perp^2} N_\alpha \frac{q_\alpha^2}{T_\alpha} (1 - \frac{\omega_\alpha^*}{\omega}) (\frac{\omega_d^\alpha}{\omega})^2 H(\zeta_\alpha) \tag{32}$$

where $H(\zeta_\alpha) = \zeta_\alpha^3 e^{-\zeta_\alpha} I(\zeta_\alpha)$, $\zeta_\alpha = \omega/\omega_d^\alpha$, and the function $I(\zeta_\alpha)$ is given by the integral:

$$I(\zeta_\alpha) = \int_0^{\sqrt{2\zeta_\alpha}} \frac{y_\perp dy_\perp e^{-\frac{y_\perp^2}{2}}}{\sqrt{\zeta_\alpha - \frac{y_\perp^2}{2}}} J_0^2(k_\perp \rho_\alpha y_\perp) \tag{33}$$

A closed expression for $I(\zeta_\alpha)$ is difficult to obtain for arbitary ζ_α. But in the limit: $k_\perp \rho_\alpha \gg 1$, we have $I(\zeta_\alpha) \simeq 1/(\sqrt{2} k_\perp \rho_\alpha)$ (see Appendix A). Since $\omega/\omega_d^\alpha \sim 1/(k_\perp \rho_\alpha)$, the linear growth rate decays as $k_\perp^{-3}$ for $k_\perp \rho_\alpha > 1$. Thus, the maximum linear growth rate occurs at $k_\perp \rho_\alpha \simeq 1$.

Now, we calculate the electron Landau damping rate. Since $k_\perp \rho_e \simeq \rho_e/\rho_\alpha \ll 1$, the electron Landau damping rate takes the same form as Eq.(31). In the normalized unit, $N_e = T_e = q_e^2 = 1$. Noting that: $\omega_e^* \ll \omega$, $\xi_e \sim \omega/k_\parallel v_e \simeq v_A/v_e \ll 1$, we have

$$\frac{\gamma_e^l}{\omega} \simeq -\frac{\sqrt{\pi}}{4k_\perp^2} \frac{v_A}{v_e} (\frac{\omega_d^e}{\omega})^2 \tag{34}$$

where $\omega_d^e = \vec{e}_\parallel \times \vec{\kappa}_c \cdot \vec{k}_\perp$ is the curvature drift frequency for thermal electrons.

b. Nonlinear Growth Rate

The nonlinear growth rate $\gamma_{\vec{k}}^{nl}$ due to ion Compton scattering can be obtained from Eqs.(15), (21), and (26). Since, $\omega \gg k_\parallel v_i$, ω_i^*, $\vec{v}_d^i \cdot \vec{k}$, we have: $R_{\vec{k},\omega}^i \simeq i/\omega$, $R_{\vec{k}',\omega'}^i \simeq i/\omega'$, but $R_{\vec{k}-\vec{k}',\omega-\omega'}^i \simeq \pi\delta(\omega' - \omega)$. Noting that $k_\perp \rho_i \ll 1$, and carrying out the velocity integration, we have (in normalized units):

$$\gamma_{\vec{k}}^{nl} = \frac{\pi}{2k_\perp^2} \sum_{\vec{k}'} (\vec{e}_\parallel \cdot \vec{k}' \times \vec{k})^2 |\tilde{\phi}|_{\vec{k}'}^2 \delta(\omega' - \omega)(\frac{\omega_s^*}{\omega} - \frac{\omega_s^{*\prime}}{\omega'}) \tag{35}$$

where ω_s^* is the ion diamagnetic frequency evaluated at electron temperature T_e. Eq.(35) has a number of important features. (1) ion Compton scattering is local in frequency ω or parallel wavenumber $k_\parallel$. (2) It can be easily verified that the nonlinear growth rate given in Eq.(35) conserves both the number of waves (or plasmons) and the wave energy, i.e.

$$\sum_{\vec{k}} \gamma_{\vec{k}}^{nl} \frac{k_\perp^2 |\tilde{\phi}|_{\vec{k}}^2}{\omega} = 0$$

$$\sum_{\vec{k}} \gamma_{\vec{k}}^{nl} k_\perp^2 |\tilde{\phi}|_{\vec{k}}^2 = 0$$

This is consistent with the approximations we made to neglect $E_\parallel$ and ion curvature drift. (3) Although ion Compton scattering conserves total fluid energy, it redistributes the fluid energy in k_θ space or induces a spectral shift. Specifically, if we denote $\langle k_\theta \rangle$ as the wavenumber at which the fluctuation spectrum peaks, then for $k_\theta < \langle k_\theta \rangle$, the mode is nonlinearly damped; while for $k_\theta > \langle k_\theta \rangle$, the mode is nonlinearly destabilized. Thus the ion Compton scattering transfers the fluctuation energies from small k_θ to high k_θ. If we denote $\bar{k}_\theta \simeq 1/\rho_\alpha$ as the wave number at which the linear growth rate is maximum, then at saturation, $\langle k_\theta \rangle > \bar{k}_\theta$. We like to emphasize that it is this spectral shift which reduces the power transfer between the alpha particles and the Alfven fluctuations that leads to the saturation of turbulence. The power transfer between the bulk electrons and Alfven wave fluctuations is virtually unaffected because the electron Landau damping rate only depends on k_θ weakly.

Now we calculate the $k'_\parallel$ summation in Eq.(35). Using $\sum_{k'_\parallel} = (L_\parallel/2\pi) \int dk'_\parallel$, where $L_\parallel$ is the size of the system in the parallel direction, we have:

$$\gamma_{\vec{k}}^{nl} = \frac{\pi}{2k_\perp^2 \omega} \frac{L_\parallel}{2\pi v_A} \sum_{\vec{k}'_\perp} (\vec{e}_\parallel \cdot \vec{k}'_\perp \times \vec{k}_\perp)^2 (\omega_s^* - \omega_s^{*\prime}) |\tilde{\phi}|^2_{\vec{k}'_\perp, k_\parallel} \tag{36}$$

In order to simplify Eq.(36), we introduce notation:

$$\langle\langle Q \rangle\rangle = \sum_{\vec{k}'_\perp} (\vec{e}_\parallel \cdot \vec{k}' \times \vec{k})^2 |\tilde{\phi}|^2_{\vec{k}'_\perp, k_\parallel} Q$$

where Q is an arbitrary function of $\vec{k}'_\perp$. If we assume that the typical parallel wavenumber is given by: $k_\parallel \simeq 2\pi/L_\parallel$, then Eq.(36) reduces to:

$$\frac{\gamma_{\vec{k}}^{nl}}{\omega} = \frac{\pi}{2k_\perp^2} \frac{\omega_s^* - \langle \omega_s^* \rangle}{\omega} \frac{I(\vec{k}_\perp, k_\parallel)}{\omega^2} \tag{37}$$

where $I(\vec{k}_\perp, k_\parallel) = \langle\langle 1 \rangle\rangle$, and $\langle \omega_i^* \rangle = \langle\langle \omega_i^* \rangle\rangle / \langle\langle 1 \rangle\rangle$.

c. Saturation Level

The wave kinetic equation which provides us with the detailed information of turbulence evolution can only be solved numerically. However, qualitative estimate of the saturation amplitude can be obtained using physical arguments. The saturation is determined by the condition that the total fluctuation energy E^f is stationary:

$$\frac{\partial}{\partial t}E^f = \sum_{\vec{k}} 2(\gamma^l_\alpha + \gamma^l_e)|\tilde{\phi}_{\vec{k}}|^2 = 0$$

where the conservation property of the ion Compton scattering has been used. Since $\gamma^l_\alpha \gg \gamma^l_e$, this stationary condition can only be achieved when the linear alpha growth rate and the wave number spectrum are completely separated in k_θ space. This implies that we need to have $\gamma^{nl}_{\vec{k}} \geq \gamma^l_{\alpha\vec{k}}$ for wavenumber $\bar{k}_\theta \simeq 1/\rho_\alpha$ at which the linear alpha growth rate $\gamma^l_{\alpha\vec{k}}$ peaks. Using Eqs.(31) and (37) for $\gamma^l_{\alpha\vec{k}}$ and $\gamma^{nl}_{\vec{k}}$ respectively, and $I(\vec{k}_\perp, k_\parallel) \simeq \bar{k}_r^2\langle k_\theta\rangle^2|\tilde{\phi}|^2$, we obtain the fluctuation level needed for saturation (in unnormalized units):

$$\frac{e\tilde{\phi}}{T_e} \simeq 0.7(\frac{N_\alpha}{N_e})^{\frac{1}{2}}(\frac{T_\alpha}{T_e})(\frac{L^i_n}{L^\alpha_n})^{\frac{1}{2}}\frac{1}{R\langle k_\theta\rangle}(1-\frac{\bar{\omega}}{\bar{\omega}^*_\alpha})^{\frac{1}{2}}(\frac{\bar{k}_\theta}{\langle k_\theta\rangle - \bar{k}_\theta})^{\frac{1}{2}} \tag{38}$$

where $\langle k_\theta\rangle = \sqrt{\langle\langle k_\theta^2\rangle\rangle/\langle\langle 1\rangle\rangle}$ is the root mean square poloidal wavenumber, R is the major radius, L^i_n and L^α_n are the radial scale lengths of bulk ions and alpha particles. The saturated magnetic fluctuation level can be obtained from $\tilde{B}_r/B_0 \simeq (c_s/v_A)(\langle k_\theta\rangle\rho_s)(e\tilde{\phi}/T_e)$.

For typical tokamak parameters like ITER[1] (International Tokamak Engineering Reactor) $R \simeq$600cm, $a \simeq$200cm, B=5T, T_e=15KeV, $T_\alpha/T_e = 50$, $N_\alpha/N_e = 5 \times 10^{-3}$, $L_n^i = a$, $L_n^\alpha = a/2$, and $\langle k_\theta \rangle - \bar{k}_\theta \sim \bar{k}_\theta$, we have the saturation level: $e\tilde{\phi}/T_e \simeq 0.03$, and $\tilde{B}_r/B_0 \simeq 10^{-4}$.

d. Particle transport

Turbulent radial particle flux can be obtained from:

$$\Gamma_r^j = \langle \int d^3\vec{v} v_r^j \tilde{f}^j \rangle_a \tag{39}$$

where $\vec{e}_r$ is a unit vector in the radial direction, $v_r^j = (c/B)\vec{e}_r \cdot \vec{e}_\parallel \times \vec{\nabla}\{\tilde{\phi} - (v_\parallel/c)\tilde{\phi}\}$ is the radial component of fluctuation induced particle drift, and $\langle \cdots \rangle_a$ is an average over the fluctuation spatial and temporal scales. In Fourier space, Eq.(39) takes the form:

$$\Gamma_r^j = -\frac{c}{B} \sum_{\vec{k}} k_\theta \int d^3\vec{v} (1 - \frac{v_\parallel k_\parallel}{\omega}) Im(\tilde{f}_{\vec{k}}^j \tilde{\phi}_{-\vec{k}}) \tag{40}$$

Radial particle flux for alpha particles, electrons and ions can be obtained by solving Eq.(1) for $\tilde{f}_{\vec{k}}^j$, and substituting it into the above equation. For the alpha particles and electrons, these particle flux are determined by the linear wave particle resonance (quasi-linear particle flux), while for the ions it is determined by the ion Compton scattering (nonlinear particle flux). The calculations are straightforward, here we only present the results (in normalized units):

$$\Gamma_r^\alpha = -\frac{2}{q_\alpha} \sum_{\vec{k}} \frac{k_\theta}{\omega_{\vec{k}}} \gamma_{\alpha \vec{k}}^l k_\perp^2 |\tilde{\phi}|_{\vec{k}}^2 \tag{41}$$

$$\Gamma_r^e = -\frac{2}{q_e}\sum_{\vec{k}}\frac{k_\theta}{\omega_{\vec{k}}}\gamma^l_{e\vec{k}}k_\perp^2|\tilde{\phi}|^2_{\vec{k}} \tag{42}$$

$$\Gamma_r^i = -\frac{2}{q_i}\sum_{\vec{k}}\frac{k_\theta}{\omega_{\vec{k}}}\gamma^{nl}_{\vec{k}}k_\perp^2|\tilde{\phi}|^2_{\vec{k}} \tag{43}$$

where the particle flux Γ_r^j is normalized to $c_s N_e$, $q_\alpha = 2$, $q_e = -1$, and $q_i = 1$ are the normalized particle charge.

We proceed to discuss the physical implications of the above equations. (1) The particle flux satisfy the ambipolarity condition at saturation, i.e.

$$\begin{aligned} q_\alpha\Gamma_r^\alpha + q_i\Gamma_r^i + q_e\Gamma_r^e &= -\sum_{\vec{k}}\frac{k_\theta}{\omega}(\gamma^l_{\alpha\vec{k}} + \gamma^l_{e\vec{k}} + \gamma^{nl}_{\vec{k}})k_\perp^2|\tilde{\phi}|^2_{\vec{k}} \\ &= 0 \end{aligned}$$

for $\gamma^l_{\alpha\vec{k}} + \gamma^l_{e\vec{k}} + \gamma^{nl}_{\vec{k}} = 0$, i.e. at saturation, there is no net radial current. (2) To see the difference between quasi-linear (alpha particle and electron) particle flux, and nonlinear (ion) particle flux, we write $\Gamma_r^i = \sum_{\vec{k}}\Gamma^i_{\vec{k}}$, where $\Gamma^i_{\vec{k}}$ is the radial particle flux induced by fluctuation at wavenumber $\vec{k}$. Since, $\gamma^l_{\alpha\vec{k}} > 0$, and $\gamma^l_{e\vec{k}} < 0$, we find $\Gamma^\alpha_{\vec{k}} > 0$, and $\Gamma^e_{\vec{k}} > 0$, i.e. both alpha particle and electron flux induced by fluctuations at all wavenumbers $\vec{k}$ are outward (down the density gradient), thus the total particle flux for alpha particles and electrons are also outward. For the ions, however, $\gamma^{nl}_{\vec{k}} > 0$ for $k_\theta > \langle k_\theta\rangle$, we have $\Gamma^i_{\vec{k}} > 0$, i.e. ion flux induced by the small scale fluctuations is outward; $\gamma^{nl}_{\vec{k}} < 0$ for $k_\theta < \langle k_\theta\rangle$, we have $\Gamma^i_{\vec{k}} < 0$, i.e. the ion flux induced by large scale fluctuations is <u>inward</u>. Since the spectral transfer is towards large k_θ, the ion flux induced by small scale fluctuations will carry more weight in the total particle flux, thus leads to a net outward ion flux. Using Eq.(35), we have:

$$\Gamma_r^i = \frac{\pi L_n^i}{2T_e} \sum_{\vec{k}} \sum_{\vec{k}'} (\vec{e}_\parallel \cdot \vec{k}' \times \vec{k})^2 |\tilde{\phi}|_{\vec{k}}^2 |\tilde{\phi}|_{\vec{k}'}^2 \delta(\omega - \omega')(\frac{\omega_s^*}{\omega} - \frac{\omega_s^{*\prime}}{\omega'})^2 > 0 \qquad (44)$$

Thus, though the alpha particles and bulk ions carry the same kind of charge (positive), the outward flow of alpha particles does not necessarily drive the bulk ions inward in order to satisfy charge neutrality condition. The outward flow of bulk ions in the radial direction is closely related to the spectral flow of fluctuation energy in k_θ space towards high k_θ. which is determined by the fact that the Alfven wave turbulence is driven by the (energetic) ions, i.e. $\omega_i^* > \omega$.

To obtain an estimate of the radial alpha particle flux, from Eq.(41) we have:

$$\Gamma_r^\alpha = \frac{2}{q_\alpha} \frac{\bar{k}_\theta}{\omega} \gamma_{\alpha\vec{k}}^l \bar{k}_\perp^2 |\tilde{\phi}|_{\langle\vec{k}\rangle}^2 \Theta \qquad (45)$$

as usual, $\bar{k}_\theta$, $\bar{k}_\perp$ are the wavenumbers where $\gamma_{\alpha\vec{k}}^l$ is maximum, $\gamma_{\alpha\vec{k}}^l$ is evaluated at $\bar{k}_\theta$, $\langle\vec{k}\rangle$ is the rms wavenumber at which the wavenumber spectrum peaks, Θ is a constant that measures the overlap between the linear alpha growth rate and wavenumber spectrum. At saturation, we should have: $\Theta \sim O(\gamma_{e\vec{k}}^l/\gamma_{\alpha\vec{k}}^l) \ll 1$ for $\gamma_{e\vec{k}}^l \ll \gamma_{\alpha\vec{k}}^l$. We define alpha diffusivity D_α by $\Gamma_r^\alpha = -D_\alpha(dN_\alpha/dr)$. Using Eq.(31) for $\gamma_{\alpha\vec{k}}^l$, and Eq.(38) for $|\tilde{\phi}|_{\langle\vec{k}\rangle}^2$, we obtain D_α (in unnormalized units):

$$\frac{D_\alpha}{c_s\rho_s} \simeq 0.5\Theta(\frac{N_\alpha}{N_e})(\frac{T_\alpha}{T_e})^2(\frac{\Omega_i}{\omega_{\vec{k}}})(\frac{\rho_s}{R})^2(\frac{L_n^i}{L_n^\alpha})(1-\frac{\omega_{\vec{k}}}{\bar{\omega}_\alpha^*})^2(\frac{\bar{k}_\theta}{\langle k_\theta\rangle - \bar{k}_\theta})(\bar{k}_\perp\rho_\alpha)^2(k_\parallel R)^{-2} \qquad (46)$$

Using the typical ITER parameters[1], and noting that $\Theta \sim \gamma^l_{e\vec{k}}/\gamma^l_{\alpha\vec{k}} \simeq 0.2$, we have: $D_\alpha \simeq 10^4 \mathrm{cm}^2/\mathrm{s}$. Electron particle flux is of the same order as the alpha particle flux as can be seen from Eqs.(42) and (45). The bulk ion particle flux is smaller because of the cancellations between large and small k_θ regions in Eq.(43).

IV. Conclusions and Discussions

In this paper, a nonlinear theory of energetic alpha particle driven Alfven wave turbulence based on ion Compton scattering is presented. It is found that ion Compton scattering, while conserving the total wave energy, induces a spectral shift in k_θ space towards high k_θ. This nonlinear wavenumber spectrum shift away from the linear alpha growth rate reduces the power transfer between the alpha particles and the Alfven wave fluctuations, thus leading to a saturation of turbulence. A saturation level of $e\tilde{\phi}/T_e \sim 0.03$ and $\tilde{B}_r/B \sim 10^{-4}$ for typical ITER-like parameters is obtained. The anomalous particle transport at steady state is also calculated. It is found that: (1) radial particle flux for both alpha particles and bulk plasmas are outward. (2) The alpha particle flux and electron particle flux are found to be of the same order because the energy balance is only determined by the alpha particle drive and electron dissipation. The ion flux is relatively smaller. The alpha particle diffusivity is calculated to be: $D^\alpha \sim 10^4 \mathrm{cm}^2/\mathrm{s}$, which is large enough to require attention in the design of future burning plasma experiment.

The results obtained in this paper indicate that the fluctuation levels and turbulent particle fluxes are not negligible. Since the calculation is based on the assumption that the nonlinear time scale is faster than the transport time scale, the question about the relative importance between the quasi-linear profile modification and ion Compton scattering in the saturation of Alfven wave turbulence is not addressed. To answer this question, studies of the coupled nonlinear evolution of the turbulence spectrum (Eq.(25)) and the alpha density profile $n_\alpha(r,t)$ are called for. A possible scenario in this case is that both processes play important roles, namely, the quasi-linear profile modification reduces the linear alpha growth rate to the point that it can be balanced by the ion Compton scattering. In this case, Eqs.(38) and (46) are still valid for an estimate of saturation level and anomalous alpha diffusivity.

Acknowledgments

The author would like to thank Profs. Sigmar, Diamond, and Rosenbluth for many helpful discussions, Dr. Hasegawa for pointing out the importance of magnetic curvature drift resonance in determining the behavior of the linear alpha growth rate in the regime of $k_\theta \rho_\alpha > 1$. This work is supported by the United States Department of Energy under contract No. DE-FG02-91ER-54109 with the Massachusetts Institute of Technology.

Appendix A: Evaluation of the Integral

In this appendix, the integral defined in Eq.(33) is evaluated. First, we note that for $k_\perp \rho_\alpha \gg 1$, $y_\perp \le \sqrt{2\zeta_\alpha} \sim 1/\sqrt{k_\perp \rho_\alpha} \ll 1$. Therefore, we can approximate $I(\zeta_\alpha)$ as:

$$I(\zeta_\alpha) = \int_0^{\sqrt{2\zeta_\alpha}} \frac{y_\perp dy_\perp}{\sqrt{\zeta_\alpha - \frac{y_\perp^2}{2}}} J_0^2(k_\perp \rho_\alpha y_\perp) \tag{A1}$$

Now we divide the integration range $[0, \sqrt{2\zeta_\alpha}]$ into two parts: one form 0 to $1/k_\perp \rho_\alpha$, the other from $1/k_\perp \rho_\alpha$ to $\sqrt{2\zeta}$. We call the integral from 0 to $1/k_\perp \rho_\alpha$ I_1, while the integral from $1/k_\perp \rho_\alpha$ to $\sqrt{2\zeta_\alpha}$ I_2, thus $I(\zeta_\alpha) = I_1(\zeta_\alpha) + I_2(\zeta_\alpha)$.

For $I_1(\zeta_\alpha)$, we have:

$$\begin{aligned} I_1(\zeta_\alpha) &= \int_0^{\frac{1}{k_\perp \rho_\alpha}} \frac{y_\perp dy_\perp}{\sqrt{\zeta_\alpha - \frac{y_\perp^2}{2}}} \\ &= 2(\sqrt{\zeta_\alpha} - \sqrt{\zeta_\alpha - \frac{1}{2}(\frac{1}{k_\perp \rho_\alpha})^2}) \end{aligned}$$

For $1/(k_\perp \rho_\alpha)^2 \ll \zeta_\alpha \sim 1/k_\perp \rho_\alpha$, we have

$$I_1(\zeta_\alpha) \simeq \frac{1}{2} \frac{1}{\sqrt{\zeta_\alpha}} \frac{1}{(k_\perp \rho_\alpha)^2} \tag{A2}$$

.

For $I_2(\zeta_\alpha)$, we note that $J_0^2(k_\perp \rho_\alpha y_\perp) \simeq [1/(\pi k_\perp \rho_\alpha y_\perp)][1 + \sin(2k_\perp \rho_\alpha y_\perp)]$, and the *sin* term make negligible contribution due to phase mixing. Thus we have:

$$\begin{aligned} I_2(\zeta_\alpha) &= \int_{\frac{1}{k_\perp \rho_\alpha}}^{\sqrt{2\zeta_\alpha}} \frac{y_\perp dy_\perp}{\sqrt{\zeta_\alpha - \frac{y_\perp^2}{2}}} \frac{1}{\pi k_\perp \rho_\alpha y_\perp} \\ &= \frac{1}{\sqrt{2} k_\perp \rho_\alpha} - \frac{1}{\pi \sqrt{\zeta_\alpha}} \frac{1}{(k_\perp \rho_\alpha)^2} \end{aligned} \tag{A3}$$

Combining Eqs.(A2) and (A3), we have

$$I(\zeta_\alpha) \simeq \frac{1}{\sqrt{\pi}} \frac{1}{k_\perp \rho_\alpha} [1 + O(\frac{1}{\sqrt{k_\perp \rho_s}})] \qquad (A4)$$

References

1. H. P. Furth, R. J. Goldston, S. J. Zweben, and D. J. Sigmar, Nucl. Fusion **30**, 1799 (1990).
2. M. N. Rosenbluth, and P. H. Rutherford, Phys. Rev. Lett. **34**, 1428, (1975).
3. A. B. Mikhailovskii, Sov. Phys. JETP **41**, 890 (1975).
4. K. T. Tsang, D. J. Sigmar, and J. C. Whitson, Phys. Fluids **24**, 1508 (1981).
5. G. Y. Fu, and J. W. Van Dam, Phys. Fluids B **1**, 1949 (1989).
6. D. J. Sigmar, and C. T. Hsu, Phys. Fluids B **4**, in press.
7. H. L. Berk, and B. N. Breizman, Phys. Fluids B **2**, 2246 (1990).
8. F. Y. Gang, D. J. Sigmar, and J. N. Leboeuf, Phys. Lett. A, 1992, in press.
9. H. Biglari, and P. H. Diamond, submitted to Phys. Rev. Lett.
10. A. Hasegawa, and L. Chen, Phys. Fluids **19**, 1924 (1976).
11. E. A. Frieman, and L. Chen, Phys. Fluids **25**, 502 (1982).
12. R. Z. Sagdeev, and A. A. Galeev, *Nonlinear Plasma Theory*, edited by T. M. O'Neil, and D. L. Brook (Benjamin, New York, 1969).
13. H. L. Berk, B. N. Breizman, and H. Ye, IFS Report No.507, the University of Texas at Austin, (1991).